

Influence of a Classical Homogeneous Gravitational Field on Dissipative Dynamics of the Phase Damped Jaynes–Cummings Model under the Markovian Approximation

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Abstract In this paper, we study the dissipative dynamics of the phase damped Jaynes–Cummings model under the Markovian approximation in the presence of a classical homogeneous gravitational field. The model consists of a moving two-level atom simultaneously exposed to the gravitational field and a single-mode traveling radiation field in the presence of a phase damping mechanism. We first present the master equation for the reduced density operator of the system under the Markovian approximation in terms of a Hamiltonian describing the atom-field interaction in the presence of a homogeneous gravitational field. Then, by making use of the super-operator technique, we obtain an exact solution of the master equation. Assuming that initially the radiation field is prepared in a Glauber coherent state and the two-level atom is in the excited state, we investigate the influence of gravity on the temporal evolution of collapses and revivals of the atomic population inversion, atomic dipole squeezing, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field in the presence of phase damping.

Keywords Jaynes–Cummings model · Atomic motion · Gravitational field · Phase damping · Non-classical properties

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1 Introduction

Over the last forty years many theoretical investigations have been addressed toward the understanding of quantum dynamics of the interacting atom-field system in a high-Q cavity. The interest toward this research area was mainly spurred by the large amount of experiments revealing the appearance of intriguing features of quantum radiation-matter interaction [1–3]. Both theoretical and experimental activities have concentrated on trying to understand simple nontrivial models of quantum optics involving a single atom, regarded as a few effective energy levels, and one or more near resonant modes of the quantized electromagnetic field. The prototype of such systems, proposed by Jaynes and Cummings in 1963, [4] describes a two-level atom resonantly interacting with a single-mode quantized field. It has proved to be a theoretical laboratory of great relevance to many topics in atomic physics and quantum optics, as well as in the ion traps [5–7], cavity QED [8, 9] and quantum information processing [10]. The Jaynes–Cummings model (JCM) also widely used in condensed matter physics for its relevance in spintronics [11] which exploits the electron-spin rather than its charge to develop a new generation of electronic devices [12, 13]. When the rotating wave approximation (RWA) is made, the model becomes exactly solvable and its dynamical features can be analytically brought to light revealing remarkable properties [14]. The discovery of interesting aspects of the JCM as well as the developments in cavity QED experiments involving single Rydberg atoms within single-mode cavities, have stimulated an intense research devoted at highlighting and generalizing the original idea and physical scenario presented by Jaynes and Cummings. In the standard JCM, the interaction between a constant electric field and a stationary (motionless) two-level atom is considered. With the development in the technologies of laser cooling and atom trapping the interaction between a moving atom and the field has attracted much attention [15–24].

Experimentally, atomic beams with very low velocities are generated in laser cooling and atomic interferometry [25]. In this situation, the influence of Earth's acceleration becomes important and cannot be neglected [26]. A semi-classical description of a two-level atom interacting with a running laser wave in a gravitational field has been studied [27, 28]. However, the semi-classical treatment does not permit us to study the pure quantum effects occurring in the course of atom-radiation interaction. Recently, within a quantum treatment of the internal and external dynamics of the atom, we have presented [29] a theoretical scheme based on an $su(2)$ dynamical algebraic structure to investigate the influence of a classical homogeneous gravitational field on the quantum non-demolition measurement of atomic momentum in the dispersive JCM. Also, we have investigated [30] quantum statistical properties of the lossless Jaynes–Cummings model in the presence of a homogeneous gravitational field. We have found that the non-classical properties are suppressed with increase of the gravitational field influence.

On the other hand, over the last two decades much attention has been focused on the properties of the dissipative variants of the JCM. The theoretical efforts have been stimulated by experimental progress in cavity QED. Besides the experimental drive, there also exists a theoretical motivation to include relevant damping mechanism to JCM because its dynamics becomes more interesting. A number of authors have treated the JCM with dissipation by the use of analytic approximations [31, 32] and numerical calculations [33–37]. The solution in the presence of dissipation is not only of theoretical interest, but also important from a practical point of view since dissipation would be always present in any experimental realization of the model. However, the dissipation treated in the above studies is modeled by coupling to an external reservoir including energy dissipation. As is well known, in a dissipative quantum system, the system loses energy by creating a bath quantum. In this

kind of damping the interaction Hamiltonian between bath and system does not commute with the system Hamiltonian. In general this leads to a thermalization of the system with a certain time constant. There are, however other kinds of environmental coupling to the system, which do not involve energy exchange. In the so-called phase damping [38, 39] the interaction Hamiltonian commutes with that of system and in the dynamics only the phase of system state is changed in the course of interaction. Similar to standard energy damping the off-diagonal elements of the density matrix in energy basis decay at a given rate. The phase damping can well describe some unaccounted decay of coherences in a single-mode micromaser [40]. It has also been shown that phase damping seriously reduces the fidelity of the received qubit in quantum computers due to the induced decoherence [41]. The phase damping in the JCM with one quantized field mode has been studied [42]. The influence of phase damping on non-classical properties of the multi-quanta two-mode JCM has also been studied [43, 44]. It has been found that the phase damping suppresses non-classical effects of the cavity field in the JCM. However, all of the foregoing studies have been done only under the condition that the influence of the gravitational field is not taken into account.

In the present contribution our main purpose is to investigate the temporal evolution of quantum statistical properties of the phase damped JCM in the presence of a classical homogeneous gravity field. By solving analytically the master equation under the Markovian approximation, the evolving reduced density operator of the system is found by which the influence of the gravitational field on the dynamical behavior of the atom-radiation system in the presence of the phase damping is explored. In Sect. 2, we present the master equation for the reduced density operator of the system under the Markovian approximation in terms of a Hamiltonian describing the atom-radiation interaction in the presence of a gravitational field. This Hamiltonian is obtained based on an $su(2)$ dynamical algebraic structure in the interaction picture. In Sect. 3, by using the super-operator technique, we obtain an exact solution of the JCM with the phase damping in the presence of a gravitational field, by which we investigate the dynamical evolution of the system. In Sect. 4 we study the influence of the gravitational field on both the cavity-field and the atomic properties in the presence of phase damping. Considering the field to be initially in a coherent state and the two-level atom in the excited state, we explore the temporal evolution of the atomic inversion, atomic dipole squeezing, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field. Finally, we summarize our conclusions in Sect. 5.

2 Master Equation for the Phase Damped JCM in the Presence of a Gravitational Field

The equation of motion for the density operator of the atom-radiation system and reservoir, $\hat{\rho}_{sr}(t)$, in the Schrödinger picture is given by [45]

$$\frac{\partial \hat{\rho}_{sr}(t)}{\partial t} = -i[\hat{H}_T, \hat{\rho}_{sr}(t)] (\hbar = 1), \quad (1)$$

where

$$\hat{H}_T = \hat{H}_s + \hat{H}_r + \hat{V}_{sr}, \quad (2)$$

with the Hamiltonian of the reservoir

$$\hat{H}_r = \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i, \quad (3)$$

and with the Hamiltonian of the interaction between the system and reservoir

$$\hat{V}_{sr} = \hat{H}_s \sum_{j=1}^3 \hat{F}_j, \tag{4}$$

where

$$\hat{F}_1 = \sum_i \kappa_i \hat{b}_i, \quad \hat{F}_2 = \sum_i \kappa_i \hat{b}_i^\dagger, \quad \hat{F}_3 = \hat{H}_s \sum_i \frac{\kappa_i^2}{2\omega_i}, \tag{5}$$

\hat{b}_i and \hat{b}_i^\dagger are the boson annihilation and creation operators for the reservoir and κ_i is the coupling constant. The Hamiltonian \hat{H}_s in (2) for the atom-radiation system in the presence of a classical gravity field with the atomic motion along the position vector \hat{x} and in the RWA is given by ($\hbar = 1$)

$$\begin{aligned} \hat{H}_s = & \frac{\hat{p}^2}{2M} - M\vec{g} \cdot \hat{x} + \omega_c \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{1}{2} \omega_{eg} \hat{\sigma}_z \\ & + \lambda [\exp(-i\vec{q} \cdot \hat{x}) \hat{a}^\dagger \hat{\sigma}_- + \exp(i\vec{q} \cdot \hat{x}) \hat{\sigma}_+ \hat{a}], \end{aligned} \tag{6}$$

where \hat{a} and \hat{a}^\dagger are, respectively, the annihilation and creation operators of a single-mode traveling wave with frequency ω_c , \vec{q} is the wave vector of the running wave and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels $|e\rangle, |g\rangle$ and Bohr transition frequency ω_{eg} . The atom-field coupling is given by the parameter λ and \hat{p}, \hat{x} denote, respectively, the momentum and position operators of the atomic center of mass motion and g is Earth’s gravitational acceleration acting on the atom. It has been shown [29] that based on an $su(2)$ algebraic structure, as the dynamical symmetry group of the model, Hamiltonian (6) takes the following form

$$\hat{H}_s = \frac{\hat{p}^2}{2M} - M\vec{g} \cdot \hat{x} + \omega_c \hat{K} + \frac{1}{2} \Delta \hat{S}_0 + \lambda \sqrt{\hat{K}} (\exp(-i\vec{q} \cdot \hat{x}) \hat{S}_- + \exp(i\vec{q} \cdot \hat{x}) \hat{S}_+), \tag{7}$$

where the operators

$$\hat{S}_0 = \frac{1}{2} (|e\rangle\langle e| - |g\rangle\langle g|), \quad \hat{S}_+ = \hat{a}|e\rangle\langle g| \frac{1}{\sqrt{\hat{K}}}, \quad \hat{S}_- = \frac{1}{\sqrt{\hat{K}}} |g\rangle\langle e| \hat{a}^\dagger, \tag{8}$$

with the following commutation relations

$$[\hat{S}_0, \hat{S}_\pm] = \pm \hat{S}_\pm, \quad [\hat{S}_-, \hat{S}_+] = -2\hat{S}_0, \tag{9}$$

are the generators of the $su(2)$ algebra, the operator $\hat{K} = \hat{a}^\dagger \hat{a} + |e\rangle\langle e|$ is a constant of motion which represents the total number of excitations of the atom-radiation and $\Delta = \omega_{eg} - \omega_c$ is the detuning parameter. The corresponding time evolution operator for Hamiltonian (7) can be expressed as [29]

$$\hat{u}(t) = \exp(iM\vec{g} \cdot \hat{x}t) \hat{v}^\dagger \hat{u}_e(t) \hat{v}, \tag{10}$$

where

$$\hat{v} = \exp(-i\vec{q} \cdot \hat{x} \hat{S}_0), \quad \hat{u}_e = \exp(-i\hat{H}'_s t). \tag{11}$$

It can be shown that the operator $\hat{u}_e(t)$ satisfies an effective Schrödinger equation governed by an effective Hamiltonian \hat{H}'_s , that is

$$i \frac{\partial \hat{u}_e}{\partial t} = \hat{H}'_s \hat{u}_e, \tag{12}$$

where

$$\begin{aligned} \hat{H}'_s = & \frac{\hat{p}^2}{2M} - \hat{\Delta}(\hat{p}, \vec{g}) \hat{S}_0 + \frac{1}{2} M g^2 t^2 + \vec{g} \cdot \hat{p} t + \lambda (\sqrt{\hat{K}} \hat{S}_- + \sqrt{\hat{K}} \hat{S}_+) \\ & + \omega_c \hat{K} - \frac{1}{2} \Delta \hat{S}_0 - \frac{q^2}{2M} \hat{S}_0 + \frac{q^2}{8M}. \end{aligned} \tag{13}$$

By using the same procedure as in [30], the Hamiltonian (13) takes the following form in the interaction picture

$$\begin{aligned} \hat{H}'_s = & \omega_c \left(\hat{a}^\dagger \hat{a} + \frac{\hat{S}_0}{2} \right) + \frac{1}{2} \hat{\Delta}(\hat{p}, \vec{g}, t) \hat{S}_0 \\ & + (\hat{k}(t) \sqrt{\hat{K}} \hat{S}_- + \hat{k}^*(t) \sqrt{\hat{K}} \hat{S}_+), \end{aligned} \tag{14}$$

where $\hat{k}(t)$ is an effective coupling coefficient

$$\hat{k}(t) = \lambda \exp\left(\frac{it}{2} \left(\hat{\Delta}(\hat{p}, t, \vec{g}) + \frac{q^2}{M} \right)\right), \tag{15}$$

and the operator

$$\hat{\Delta}(\hat{p}, t, \vec{g}) = \omega_c - \left(\omega_{eg} + \frac{\vec{q} \cdot \hat{p}}{M} + \vec{q} \cdot \vec{g} t + \frac{q^2}{2M} \right), \tag{16}$$

has been introduced as the Doppler shift detuning at time t [29]. The Hamiltonian (14) has the form of the Hamiltonian of the JCM, the only modification being the dependence of the detuning on the conjugate momentum and the gravitational field. Now according to ref. [29] we consider

$$\hat{\rho}_{sr}(t) = \exp(iM\vec{g} \cdot \hat{x}t) \hat{v}^\dagger (\hat{\rho}_{sr})_e(t) \hat{v}, \quad \hat{v} = \exp(-i\vec{q} \cdot \hat{x} \hat{S}_0). \tag{17}$$

It can be shown that the operator $(\hat{\rho}_{sr})_e(t)$ satisfies an effective Schrödinger equation governed by an effective Hamiltonian $\hat{H}'_s = \hat{H}'_s + \hat{H}_r + \hat{V}_{sr}$, that is

$$i \frac{\partial (\hat{\rho}_{sr})_e(t)}{\partial t} = \hat{H}'_s (\hat{\rho}_{sr})_e(t). \tag{18}$$

By using the method given in [29] we obtain the following equation of motion in the interaction picture

$$\frac{\partial \hat{\rho}'_{sr}(t)}{\partial t} = -i [\hat{H}'_s, \hat{\rho}'_{sr}(t)]. \tag{19}$$

The master equation for the reduced density operator of the atom-radiation system, $\hat{\rho}'_s(t) = \text{Tr}_r \hat{\rho}'_{sr}(t)$, under the Markovian approximation with neglecting the Lamb-shift term reads as [45]

$$\frac{\partial \hat{\rho}'_s(t)}{\partial t} = -i \gamma [\hat{H}'_s, [\hat{H}'_s, \hat{\rho}'_s(t_0)]]. \tag{20}$$

Now, we consider

$$\hat{\rho}_s(t) = \hat{U}_1^\dagger \hat{\rho}_s^I \hat{U}_1, \tag{21}$$

with

$$\hat{U}_1 = \mathcal{T} \left\{ \exp \left(i \int_0^t \hat{H}_s^I(t') dt' \right) \right\}, \tag{22}$$

where the symbol \mathcal{T} denotes time ordering. By using (14), (20), (21) and (22), and following the same procedure as in ref. [45] we obtain the master equation for the reduced density operator of the system under Markovian approximation with neglecting the lamb shift term

$$\frac{\partial \hat{\rho}_s(t)}{\partial t} = -i[\hat{H}_s^I, \hat{\rho}_s(t)] - \gamma[\hat{H}_s^I, [\hat{H}_s^I, \hat{\rho}_s(t)]], \tag{23}$$

where \hat{H}_s^I is given by (14). In (23), γ is a parameter which depends on the temperature T

$$\gamma = \Delta\omega' + 2\pi T \lim_{\omega \rightarrow 0} \left(\frac{J(\omega)|\kappa(\omega)|^2}{\omega} \right), \tag{24}$$

where

$$\Delta\omega' = i \int_0^\infty d\omega J(\omega) \frac{|\kappa(\omega)|^2}{\omega}, \tag{25}$$

and $J(\omega)$ and $\kappa(\omega)$ are the spectral density of the reservoir and the coupling coefficient, respectively. In the derivation of the master equation we have assumed that the parameter T is high enough so that the Markovian approximation is valid.

3 Dynamical Evolution of the Phase Damped JCM in the Presence of Classical Gravity

In Sect. 2, we obtained the master equation for the reduced density operator of the atom-radiation system under the Markovian approximation in the presence of a classical homogeneous gravitational field. In this section, We now start to find the exact solution for the density operator $\hat{\rho}_s(t)$ of the master equation (23) with the Hamiltonian (14). For this purpose, we apply the approach presented in Refs. [46–48]. The formal solution is given by

$$\hat{\rho}_s(t) = \exp(\hat{R}t) \exp(\hat{S}t) \exp(\hat{T}t) \hat{\rho}_s(0), \tag{26}$$

where $\hat{\rho}_s(0)$ is the density operator of the initial atom-field system. The auxiliary super-operators \hat{R} , \hat{S} and \hat{T} are defined through their action on the density operator such that

$$\exp(\hat{R}t) \hat{\rho}_s(0) \equiv \sum_{k=0}^\infty \frac{(2\gamma t)^k}{k!} (\hat{H}_s^I)^k \hat{\rho}_s(0) (\hat{H}_s^I)^k, \tag{27}$$

$$\exp(\hat{S}t) \hat{\rho}_s(0) \equiv \exp(-i \hat{H}_s^I t) \hat{\rho}_s(0) \exp(i \hat{H}_s^I t), \tag{28}$$

$$\exp(\hat{T}t) \hat{\rho}_s(0) \equiv \exp(-\gamma (\hat{H}_s^I)^2 t) \hat{\rho}_s(0) \exp(-\gamma (\hat{H}_s^I)^2 t). \tag{29}$$

We assume that initially the radiation field is in a coherent superposition of the Fock states, the atom is in the excited state $|e\rangle$, and the state vector for the center-of-mass degree

of freedom is $|\psi_{c.m}(0)\rangle = \int d^3 p \phi(\vec{p})|\vec{p}\rangle$. Therefore, the initial density operator of the atom-radiation system reads as

$$\hat{\rho}_s(0) = \hat{\rho}_{\text{field}}(0) \otimes \hat{\rho}_{\text{atom}}(0) \otimes \hat{\rho}_{c.m}(0) = \begin{bmatrix} \hat{\rho}_{\text{field}}(0) \otimes \hat{\rho}_{c.m}(0) & 0 \\ 0 & 0 \end{bmatrix}, \tag{30}$$

where

$$\hat{\rho}_{\text{field}}(0) = \sum_n \sum_m w_n(0)w_m(0)|n\rangle\langle m|, \tag{31}$$

$$\hat{\rho}_{c.m}(0) = \int d^3 p \int d^3 p' \phi^*(\vec{p}')\phi(\vec{p})|\vec{p}\rangle\langle\vec{p}'|, \tag{32}$$

with $w_n(0) = \frac{\exp(-\frac{|\alpha|^2}{2})\alpha^n}{\sqrt{n!}}$. The Hamiltonian (14) can be expressed as a sum of two terms which commute with each other, that is,

$$\hat{H}'_s = \hat{H}_1 + \hat{H}_2, [\hat{H}_1, \hat{H}_2] = 0 \tag{33}$$

where

$$\hat{H}_1 = \omega_c \left(\hat{a}^\dagger \hat{a} + \frac{\hat{S}_0}{2} \right), \tag{34}$$

$$\hat{H}_2 = \frac{1}{2} \hat{\Delta}(\vec{p}, \vec{g}, t) \hat{S}_0 + (\hat{\kappa}(t) \sqrt{\hat{K}} \hat{S}_- + \hat{\kappa}^*(t) \sqrt{\hat{K}} \hat{S}_+). \tag{35}$$

In the two-dimensional atomic basis we have

$$\hat{H}_1 = \omega_c \begin{bmatrix} \hat{n} + \frac{1}{2} & 0 \\ 0 & \hat{n} - \frac{1}{2} \end{bmatrix}, \tag{36}$$

$$\hat{H}_2 = \begin{bmatrix} \frac{\Delta(\vec{p}, \vec{g}, t)}{4} & \kappa^*(t) \hat{a} \\ \kappa(t) \hat{a}^\dagger & -\frac{\Delta(\vec{p}, \vec{g}, t)}{4} \end{bmatrix}. \tag{37}$$

Also, the square of the Hamiltonian (14) can be expressed as a sum of two operators, one of them is diagonal, in the form

$$(\hat{H}'_s)^2 = \hat{A}_1 + \hat{A}_2, \tag{38}$$

where

$$\begin{aligned} \hat{A}_1 &= \hat{H}_1^2 + \hat{H}_2^2 \\ &= \begin{bmatrix} \omega_c^2(\hat{n} + \frac{1}{2})^2 + \lambda^2(\hat{n} + 1) + (\frac{\Delta(\vec{p}, \vec{g}, t)}{4})^2 & 0 \\ 0 & \omega_c^2(\hat{n} - \frac{1}{2})^2 + \lambda^2\hat{n} + (\frac{\Delta(\vec{p}, \vec{g}, t)}{4})^2 \end{bmatrix}, \end{aligned} \tag{39}$$

and

$$\hat{A}_2 = 2\hat{H}_1\hat{H}_2 = 2\omega_c \begin{bmatrix} (\hat{n} + \frac{1}{2})(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}) & (\hat{n} + \frac{1}{2})\kappa^*(t)\hat{a} \\ (\hat{n} - \frac{1}{2})\kappa(t)\hat{a}^\dagger & -(\hat{n} - \frac{1}{2})(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}) \end{bmatrix}. \tag{40}$$

It is easily proved that $[\hat{A}_1, \hat{A}_2] = 0$. Taking into account the initial condition (30) we define the auxiliary density operator $\hat{\rho}_2(t)$ as

$$\begin{aligned} \hat{\rho}_2(t) &= \exp(\hat{S}t) \exp(\hat{T}t) \hat{\rho}_s(0) \\ &= \exp(-i\hat{H}_2t) \exp(-\gamma\hat{A}_2t) \hat{\rho}_1(t) \exp(-\gamma\hat{A}_2t) \exp(i\hat{H}_2t), \end{aligned} \tag{41}$$

where the operator $\hat{\rho}_1(t)$ is defined by

$$\hat{\rho}_1(t) = |\Psi(t)\rangle\langle\Psi(t)| \otimes |e\rangle\langle e|, \tag{42}$$

with

$$|\Psi(t)\rangle = \exp\left(-\gamma t \left[\omega_c^2 \left(\hat{n} + \frac{1}{2}\right)^2 + \lambda^2(\hat{n} + 1) + \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 \right]\right) w_n(0) \exp(-in\omega_c t) |n\rangle. \tag{43}$$

From (36) and (39) we have, respectively

$$\exp(-i\hat{H}_1t) = \begin{bmatrix} \exp(-i\omega_c(\hat{n} + \frac{1}{2})) & 0 \\ 0 & \exp(-i\omega_c(\hat{n} - \frac{1}{2})) \end{bmatrix}, \tag{44}$$

$$\exp(-\gamma\hat{A}_1t) = \begin{bmatrix} (\hat{A}_1)_{11}(\hat{n}, t) & 0 \\ 0 & (\hat{A}_1)_{22}(\hat{n}, t) \end{bmatrix}, \tag{45}$$

where

$$(\hat{A}_1)_{11}(\hat{n}, t) = \exp\left(-\gamma t \left[\omega_c^2 \left(\hat{n} + \frac{1}{2}\right)^2 + \lambda^2(\hat{n} + 1) + \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 \right]\right), \tag{46}$$

$$(\hat{A}_1)_{22}(\hat{n}, t) = \exp\left(-\gamma t \left[\omega_c^2 \left(\hat{n} - \frac{1}{2}\right)^2 + \lambda^2\hat{n} + \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 \right]\right). \tag{47}$$

Also, we can write

$$\exp(-\gamma\hat{A}_2t) = \begin{bmatrix} \hat{e}_1(\hat{n}, t) & \hat{e}_2(\hat{n}, t)\hat{a} \\ \hat{e}_3(\hat{n}, t)\hat{a}^\dagger & \hat{e}_4(\hat{n}, t) \end{bmatrix}, \tag{48}$$

where

$$\hat{e}_1(\hat{n}, t) = \cos(\gamma t \sqrt{\hat{c}_1(\hat{n}, t)}) - \omega_c \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{2}\right) \left(\hat{n} + \frac{1}{2}\right) \frac{\sinh(\gamma t \sqrt{\hat{c}_1(\hat{n}, t)})}{\sqrt{\hat{c}_1(\hat{n}, t)}}, \tag{49}$$

$$\hat{e}_2(\hat{n}, t) = -2\omega_c \lambda \left(\hat{n} - \frac{1}{2}\right) \frac{\sinh(\gamma t \sqrt{\hat{c}_1(\hat{n} - 1, t)})}{\sqrt{\hat{c}_1(\hat{n} - 1, t)}}, \tag{50}$$

$$\hat{e}_3(\hat{n}, t) = -2\omega_c \lambda \left(\hat{n} - \frac{1}{2}\right) \frac{\sinh(\gamma t \sqrt{\hat{c}_2(\hat{n}, t)})}{\sqrt{\hat{c}_2(\hat{n}, t)}}, \tag{51}$$

$$\hat{e}_4(\hat{n}, t) = \cos(\gamma t \sqrt{\hat{c}_2(\hat{n}, t)}) - \omega_c \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{2}\right) \left(\hat{n} - \frac{1}{2}\right) \frac{\sinh(\gamma t \sqrt{\hat{c}_2(\hat{n}, t)})}{\sqrt{\hat{c}_2(\hat{n}, t)}}, \tag{52}$$

with

$$\hat{c}_1(\hat{n}, t) = \omega_c^2 \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{2} \right)^2 \left(\hat{n} + \frac{1}{2} \right)^2 + \lambda^2 \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{2} \right)^2 (\hat{n} + 1) \left(\hat{n} + \frac{1}{2} \right)^2, \tag{53}$$

$$\hat{c}_2(\hat{n}, t) = \omega_c^2 \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{2} \right)^2 \left(\hat{n} - \frac{1}{2} \right)^2 + \lambda^2 \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{2} \right)^2 \hat{n} \left(\hat{n} - \frac{1}{2} \right)^2. \tag{54}$$

Similarly, we can express the operator $\exp(-i\hat{H}_2t)$ in the two-dimensional atomic basis as

$$\exp(-i\hat{H}_2t) = \begin{bmatrix} \hat{d}_1(\hat{n}, t) & \hat{d}_2(\hat{n}, t)\hat{a} \\ \hat{d}_3(\hat{n}, t)\hat{a}^\dagger & \hat{d}_4(\hat{n}, t) \end{bmatrix}, \tag{55}$$

where

$$\begin{aligned} \hat{d}_1(\hat{n}, t) &= \cos\left(t\left(\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2(\hat{n} + 1)\right)\right) \\ &\quad - \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right) \frac{\sin\left(t\left(\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2(\hat{n} + 1)\right)\right)}{\sqrt{\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2(\hat{n} + 1)}}, \end{aligned} \tag{56}$$

$$\hat{d}_2(\hat{n}, t) = -i\lambda \frac{\sin\left(t\left(\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2(\hat{n} + 1)\right)\right)}{\sqrt{\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2(\hat{n} + 1)}}, \tag{57}$$

$$\hat{d}_3(\hat{n}, t) = -i\lambda \frac{\sin\left(t\left(\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2\hat{n}\right)\right)}{\sqrt{\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2\hat{n}}}, \tag{58}$$

$$\hat{d}_4(\hat{n}, t) = \cos\left(t\left(\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2\hat{n}\right)\right) - \left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right) \frac{\sin\left(t\left(\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2\hat{n}\right)\right)}{\sqrt{\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4}\right)^2 + \lambda^2\hat{n}}}. \tag{59}$$

Then, from (48) and (55), it follows that

$$\exp(-i\hat{H}_2t) \exp(-\gamma\hat{A}_2t) = \begin{bmatrix} \hat{f}_1(\hat{n}, t) & \hat{f}_2(\hat{n}, t)\hat{a} \\ \hat{f}_3(\hat{n}, t)\hat{a}^\dagger & \hat{f}_4(\hat{n}, t) \end{bmatrix}, \tag{60}$$

where

$$\hat{f}_1(\hat{n}, t) = \hat{e}_1(\hat{n}, t)\hat{d}_1(\hat{n}, t) + \hat{e}_2(\hat{n}, t)\hat{d}_2(\hat{n}, t), \tag{61}$$

$$\hat{f}_2(\hat{n}, t) = \hat{e}_2(\hat{n}, t)\hat{d}_1(\hat{n}, t) + \hat{e}_1(\hat{n}, t)\hat{d}_2(\hat{n}, t), \tag{62}$$

$$\hat{f}_3(\hat{n}, t) = \hat{e}_3(\hat{n}, t)\hat{d}_4(\hat{n}, t) + \hat{e}_4(\hat{n}, t)\hat{d}_3(\hat{n}, t), \tag{63}$$

$$\hat{f}_4(\hat{n}, t) = \hat{e}_4(\hat{n}, t)\hat{d}_4(\hat{n}, t) + \hat{e}_3(\hat{n}, t)\hat{d}_3(\hat{n}, t). \tag{64}$$

Substituting (42) and (60) into (41), we can obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows

$$(\hat{\rho}_2(t))_{i,j} = |\Psi_i(t)\rangle\langle\Psi_j(t)| \quad (i, j = 1, 2), \tag{65}$$

with

$$|\Psi_1(t)\rangle = \hat{f}_1(\hat{n}, t)|\Psi(t)\rangle, \quad |\Psi_2(t)\rangle = \hat{f}_3(\hat{n}, t)|\Psi(t)\rangle, \tag{66}$$

where $|\Psi(t)\rangle$ is given by (43). Now, we obtain the action of the operator $\exp(\hat{R}t)$ on the operator $\hat{\rho}_2(t)$

$$\hat{\rho}_s(t) = \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{H}^k \hat{\rho}_2(t) \hat{H}^k, \tag{67}$$

where

$$\hat{H}^k = \sum_{l=0}^k \frac{k!}{l!(k-l)!} \hat{H}_1^{k-l} \hat{H}_2^l, \tag{68}$$

which can be explicitly expressed as follows

$$\hat{H}^k = \begin{bmatrix} \hat{g}_+^k(\hat{n}, t) & \kappa^*(t) \frac{\hat{u}_-^k(\hat{n}, t)}{\sqrt{(\frac{\Delta(\vec{p}, \vec{g}, t)}{4})^2 + \lambda^2(\hat{n}+1)}} \hat{a} \\ \kappa(t) \frac{\hat{v}_-^k(\hat{n}, t)}{\sqrt{(\frac{\Delta(\vec{p}, \vec{g}, t)}{4})^2 + \lambda^2(\hat{n}+1)}} \hat{a}^\dagger & \hat{g}_-^k(\hat{n}, t) \end{bmatrix}, \tag{69}$$

where

$$\hat{g}_+^k(\hat{n}, t) = \hat{u}_+^k(\hat{n}, t) + \frac{\Delta(\vec{p}, \vec{g}, t)}{4} \hat{u}_-^k(\hat{n}, t), \tag{70}$$

$$\hat{g}_-^k(\hat{n}, t) = \hat{v}_+^k(\hat{n}, t) - \frac{\Delta(\vec{p}, \vec{g}, t)}{4} \hat{v}_-^k(\hat{n}, t), \tag{71}$$

$$\hat{u}_\pm^k(\hat{n}, t) = \frac{1}{2}(\hat{r}_+^k(\hat{n}, t) \pm \hat{r}_-^k(\hat{n}, t)), \quad \hat{v}_\pm^k(\hat{n}, t) = \frac{1}{2}(\hat{s}_+^k(\hat{n}, t) \pm \hat{s}_-^k(\hat{n}, t)), \tag{72}$$

with

$$\hat{r}_\pm(\hat{n}, t) = \omega_c \left(\hat{n} + \frac{1}{2} \right) \pm \sqrt{\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4} \right)^2 + \lambda^2(\hat{n} + 1)}, \tag{73}$$

$$\hat{s}_\pm(\hat{n}, t) = \omega_c \left(\hat{n} - \frac{1}{2} \right) \pm \sqrt{\left(\frac{\Delta(\vec{p}, \vec{g}, t)}{4} \right)^2 + \lambda^2\hat{n}}. \tag{74}$$

Finally, by substituting (65) and (69) into (67) we obtain the exact solution of the master equation (23) for the phase damped JCM in the presence of a classical homogeneous gravity field

$$\hat{\rho}_s(t) = \begin{bmatrix} \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{M}_{11}^k(t) & \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{M}_{12}^k(t) \\ \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{M}_{21}^k(t) & \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{M}_{22}^k(t) \end{bmatrix}, \tag{75}$$

where

$$\begin{aligned} \hat{M}_{11}^k(t) &= (\hat{g}_+^k(\hat{n}, t) \hat{\Psi}_{11}(t) \hat{g}_+^k(\hat{n}, t) + \hat{a} \hat{v}_-^{k'}(\hat{n}, t) \hat{\Psi}_{21}(t) \hat{g}_+^k(\hat{n}, t) \\ &+ \hat{g}_+^k(\hat{n}, t) \hat{\Psi}_{12}(t) \hat{v}_-^{k'}(\hat{n}, t) \hat{a}^\dagger + \hat{a} \hat{v}_-^{k'}(\hat{n}, t) \hat{\Psi}_{22}(t) \hat{v}_-^{k'}(\hat{n}, t) \hat{a}^\dagger) |\phi(\vec{p})|^2, \end{aligned} \tag{76}$$

$$\begin{aligned} \hat{M}_{22}^k(t) &= (\hat{v}'_-(\hat{n}, t)\hat{a}^\dagger\hat{\Psi}_{11}(t)\hat{a}\hat{v}'_-(\hat{n}, t) + \hat{g}'_-(\hat{n}, t)\hat{\Psi}_{21}(t)\hat{a}\hat{v}'_+(\hat{n}, t) \\ &\quad + \hat{v}'_+(\hat{n}, t)\hat{a}^\dagger\hat{\Psi}_{12}(t)\hat{g}'_-(\hat{n}, t) + \hat{g}'_-(\hat{n}, t)\hat{\Psi}_{22}(t)\hat{g}'_-(\hat{n}, t))|\phi(\vec{p})|^2, \end{aligned} \tag{77}$$

$$\begin{aligned} \hat{M}_{21}^k(t) &= (\hat{M}_{12}^k(t))^\dagger \\ &= (\hat{v}'_-(\hat{n}, t)\hat{a}^\dagger\hat{\Psi}_{11}(t)\hat{g}'_+(\hat{n}, t) + \hat{g}'_-(\hat{n}, t)\hat{\Psi}_{21}(t)\hat{a}\hat{g}'_+(\hat{n}, t) \\ &\quad + \hat{v}'_+(\hat{n}, t)\hat{a}^\dagger\hat{\Psi}_{12}(t)\hat{v}'_-(\hat{n}, t)\hat{a}^\dagger + \hat{g}'_-(\hat{n}, t)\hat{\Psi}_{22}(t)\hat{v}'_-(\hat{n}, t)\hat{a}^\dagger)|\phi(\vec{p})|^2, \end{aligned} \tag{78}$$

with

$$\hat{v}'_{\pm}^k(\hat{n}, t) = \frac{\lambda}{\sqrt{(\frac{\Delta(\vec{p}, \vec{g}, t)}{4})^2 + \lambda^2 \hat{n}}} \hat{v}_{\pm}^k(\hat{n}, t). \tag{79}$$

Making use of the solution given by (75), one can evaluate the mean values of operators of interest. In the next section we shall use it to investigate various dynamical properties of the phase damped JCM in the presence of a homogeneous gravitational field.

4 Dynamical Properties

In this section, we study the influence of the gravitational field on the quantum statistical properties of the atom and the quantized radiation field in the presence of phase damping.

4.1 Atomic Population Inversion

The atomic inversion is defined as the difference between the probabilities to find the atom in the excited state and the ground state,

$$W(t) = \langle \hat{\sigma}_3(t) \rangle = \text{Tr}_{\text{atom}}(\hat{\rho}_{\text{atom}}(t)\hat{\sigma}_3(t)), \tag{80}$$

where

$$\hat{\rho}_{\text{atom}}(t) = \text{Tr}_{\text{field}}(\hat{\rho}_s(t)). \tag{81}$$

We can rewrite (80) as follows

$$\begin{aligned} W(t) &= \int d^3 p \sum_{i=e,g} \langle i|\hat{\rho}_{\text{atom}}(t)\hat{\sigma}_3(t)|i\rangle \\ &= \int d^3 p \sum_{n=0}^{\infty} (\langle n|\otimes (\langle e|\hat{\rho}_s(t)|e\rangle - \langle g|\hat{\rho}_s(t)|g\rangle) \otimes |n\rangle). \end{aligned} \tag{82}$$

Therefore, by using (75) and (82) we obtain

$$W(t) = \int d^3 p \left(\sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n|\hat{M}_{11}^k(t)|n\rangle - \langle n|\hat{M}_{22}^k(t)|n\rangle) \right), \tag{83}$$

where from (76) and (77) we have

$$\begin{aligned} \langle n|\hat{M}_{11}^k(t)|n\rangle &= (g_+^k(n, t))^2 |\psi_1(n, t)|^2 + (h^k(n + 1, t))^2 |\psi_2(n + 1, t)|^2 \\ &\quad + 2\text{Re}[g_+^k(n, t)h^k(n + 1, t)\psi_1^*(n, t)\psi_2(n + 1, t)], \end{aligned} \tag{84}$$

$$\begin{aligned} \langle n | \hat{M}_{22}^k(t) | n \rangle &= (h^k(n, t))^2 |\psi_1(n - 1, t)|^2 + (g_-^k(n, t))^2 |\psi_2(n, t)|^2 \\ &+ 2 \operatorname{Re}[g_-^k(n, t) h^k(n, t) \psi_1^*(n - 1, t) \psi_2(n, t)], \end{aligned} \tag{85}$$

with

$$h^k(n, t) = \sqrt{n} v_-^k(n, t), \tag{86}$$

and

$$\psi_i(n, t) = \langle n | \Psi_i(t) \rangle \quad (i = 1, 2), \tag{87}$$

where we have defined $|\Psi_i(t)\rangle$ in (66).

In Fig. 1 we have plotted the atomic population inversion as a function of the scaled time λt for three different values of the parameter $\vec{q} \cdot \vec{g}$. In this figure and all the subsequent figures we set $q = 10^7 \text{ m}^{-1}$, $M = 10^{-26} \text{ Kg}$, $g = 9.8 \frac{\text{m}}{\text{s}^2}$, $\omega_{\text{rec}} = \frac{q^2}{2M} = 0.5 \times 10^6 \frac{\text{rad}}{\text{s}}$, $\lambda = 1 \times 10^6 \frac{\text{rad}}{\text{s}}$, $\Delta_0 = 8.5 \times 10^7 \frac{\text{rad}}{\text{s}}$, $\alpha = 2$, $\Delta = 1.8 \times 10^6 \frac{\text{rad}}{\text{s}}$, $\phi(\vec{p}) = \frac{1}{\sqrt{2\pi\sigma_0}} \exp(-\frac{p^2}{\sigma_0^2})$ with $\sigma_0 = 1$ [27–29] and $\gamma = 7 \times 10^{-5} \frac{\text{rad}}{\text{s}}$. It should be noted that the relevant time scale introduced by the gravitational influence is $\tau_a = \frac{1}{\sqrt{q \cdot g}}$ [29]. For an optical laser with $q = 10^7 \text{ m}^{-1}$, τ_a is about 10^{-4} s . In Fig. 1a we consider small gravitational influence in the presence of the phase damping. This means very small $\vec{q} \cdot \vec{g}$, i.e., the momentum transfer from the laser beam to the atom is only slightly altered by the gravitational acceleration because the latter is very small or nearly perpendicular to the laser beam. In Figs. 1b and c we consider the gravitational influence in the presence of the phase damping for $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$ and $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$, respectively. By comparing Figs. 1a, b and c we can see the influence of gravity on the time evolution of the atomic population inversion when there is the phase damping. As it is seen from Fig. 1a for the atomic population inversion the Rabi-like oscillations can be identified. With the increasing value of the parameter $\vec{q} \cdot \vec{g}$ (see Figs. 1b and c) the Rabi oscillations of the atomic population inversion disappear.

4.2 Atomic Dipole Squeezing

To analyze the quantum fluctuations of the atomic dipole variables and examine their squeezing we consider the two slowly varying Hermitian quadrature operators

$$\hat{\sigma}_1 = \frac{1}{2} (\hat{\sigma}_+ \exp(-i\omega_{eg}t) + \hat{\sigma}_- \exp(i\omega_{eg}t)), \tag{88}$$

and

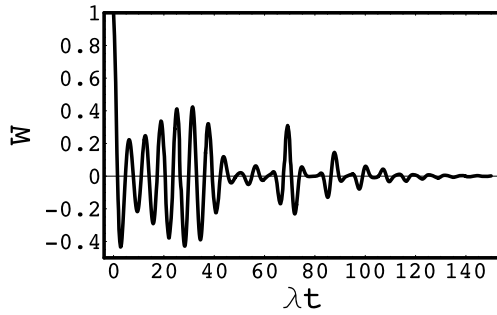
$$\hat{\sigma}_2 = \frac{1}{2i} (\hat{\sigma}_+ \exp(-i\omega_{eg}t) - \hat{\sigma}_- \exp(i\omega_{eg}t)). \tag{89}$$

In fact $\hat{\sigma}_1$ and $\hat{\sigma}_2$ correspond to the dispersive and absorptive components of the amplitude of the atomic polarization [49–51], respectively. They obey the commutation relation $[\hat{\sigma}_1, \hat{\sigma}_2] = \frac{i}{2} \hat{\sigma}_3$. Correspondingly, the Heisenberg uncertainty relation is

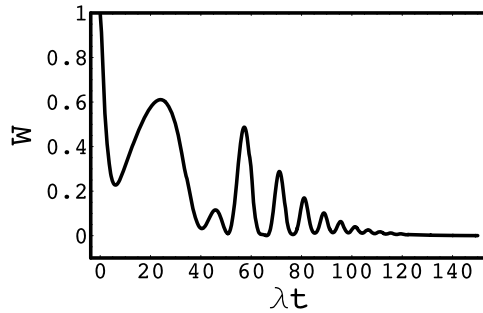
$$(\Delta \hat{\sigma}_1)^2 (\Delta \hat{\sigma}_2)^2 \geq \frac{1}{16} |\langle \hat{\sigma}_3 \rangle|^2, \tag{90}$$

where $(\Delta \hat{\sigma}_i)^2 = \langle \hat{\sigma}_i^2 \rangle - \langle \hat{\sigma}_i \rangle^2$ is the variance in the component $\hat{\sigma}_i$ ($i = 1, 2$) of the atomic dipole.

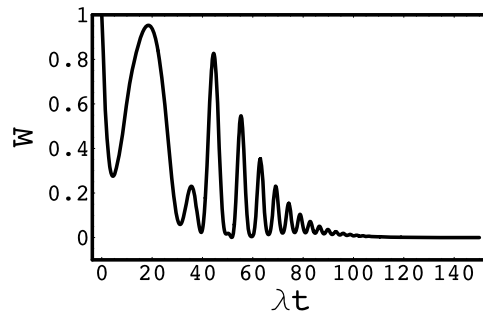
Fig. 1 Time evolution of the atomic population inversion versus the scaled time λt . Here we have set $q = 10^7 \text{ m}^{-1}$, $M = 10^{-26} \text{ kg}$, $g = 9.8 \frac{\text{m}}{\text{s}^2}$, $\omega_{\text{rec}} = 0.5 \times 10^6 \frac{\text{rad}}{\text{s}}$, $\lambda = 1 \times 10^6 \frac{\text{rad}}{\text{s}}$, $\Delta_0 = 8.5 \times 10^7 \frac{\text{rad}}{\text{s}}$, $\varphi = 0$, $\alpha = 2$, $\Delta = 1.8 \times 10^6 \frac{\text{rad}}{\text{s}}$, $\gamma = 7 \times 10^{-5} \frac{\text{rad}}{\text{s}}$, **a** For $\vec{q} \cdot \vec{g} = 0$; **b** For $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$; **c** For $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$



(a)



(b)



(c)

The fluctuations in the component $\hat{\sigma}_i$ ($i = 1, 2$) are said to be squeezed (i.e., dipole squeezing) if the variance in $\hat{\sigma}_i$ satisfies the condition

$$(\Delta \hat{\sigma}_i)^2 < \frac{1}{4} |\langle \hat{\sigma}_3 \rangle| \quad (i = 1 \text{ or } 2). \tag{91}$$

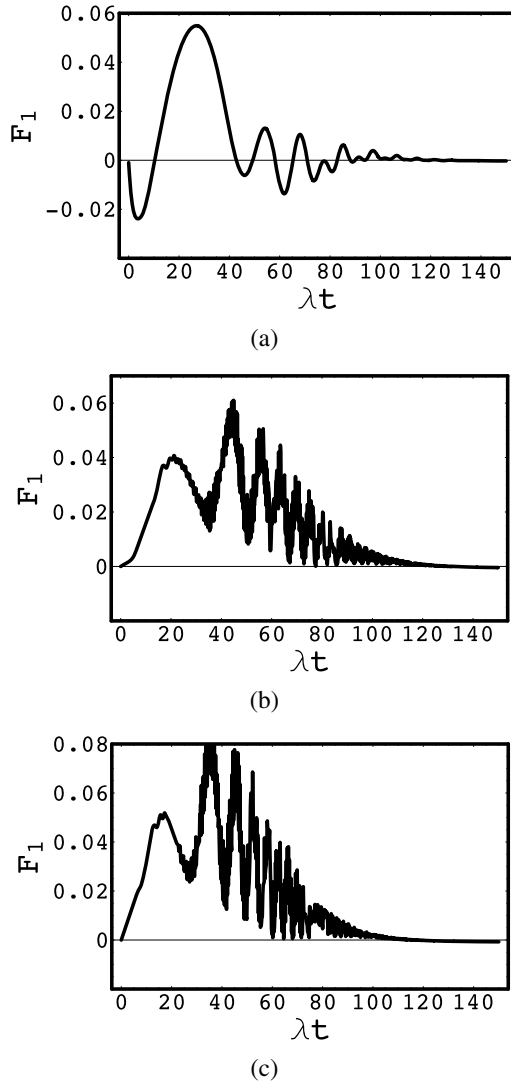
Since $\hat{\sigma}_i^2 = \frac{1}{4}$ this condition may be written as

$$F_i = 1 - 4 \langle \hat{\sigma}_i \rangle^2 - |\langle \hat{\sigma}_3 \rangle| < 0 \quad (i = 1 \text{ or } 2). \tag{92}$$

The expectation values of the atomic operators $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are given by

$$\langle \hat{\sigma}_{\pm}(t) \rangle = \text{Tr}_{\text{atom}}(\hat{\rho}_{\text{atom}}(t) \hat{\sigma}_{\pm}(t)). \tag{93}$$

Fig. 2 Time evolution of the atomic dipole squeezing versus the scaled time λt with the same corresponding data used in Fig. 1: **a** For $\vec{q} \cdot \vec{g} = 0$; **b** For $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$; **c** For $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$



Therefore, by using (75) and (81) we obtain

$$\langle \hat{\sigma}_-(t) \rangle = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} \langle n | \hat{M}_{12}^k(t) | n \rangle = \langle \hat{\sigma}_+(t) \rangle^*, \tag{94}$$

where from (78) we have

$$\begin{aligned} \langle n | \hat{M}_{12}^k(t) | n \rangle &= g_+^k(n, t) \psi_1(n, t) (g_-^k(n, t) \psi_2^*(n, t) + h^k(n, t) \psi_1^*(n - 1, t)) \\ &+ h^k(n + 1, t) \psi_2(n + 1, t) (h^k(n, t) \psi_1^*(n - 1, t) + g_-^k(n, t) \psi_2^*(n, t)). \end{aligned} \tag{95}$$

The time evolution of $F_1(t)$ corresponding to the squeezing of $\hat{\sigma}_1$ has been shown in Fig. 2 for three values of the parameter $\vec{q} \cdot \vec{g}$ in the presence of the phase damping. As it

is seen, with the increasing value of the parameter $\vec{q} \cdot \vec{g}$ the dipole squeezing is completely removed.

4.3 Atomic Momentum Diffusion

The next quantity we examine is the atomic momentum diffusion. As a consequence of the atomic momentum diffusion, the atom experiences light-induced forces (radiation force) during its interaction with the radiation field. The atomic momentum diffusion is given by

$$\Delta p(t) = (\langle \hat{p}(t)^2 \rangle - \langle \hat{p}(t) \rangle^2)^{\frac{1}{2}}. \tag{96}$$

Now we calculate the expectation values of the \hat{p} and \hat{p}^2

$$\langle \hat{p}(t) \rangle = \text{Tr}_{\text{atom}}(\hat{\rho}_{\text{atom}}(t)\hat{p}(t)), \tag{97}$$

$$\langle \hat{p}(t)^2 \rangle = \text{Tr}_{\text{atom}}(\hat{\rho}_{\text{atom}}(t)\hat{p}(t)^2). \tag{98}$$

By using $\hat{p}|p\rangle = p|p\rangle$ and (75) we obtain

$$\langle \hat{p}(t) \rangle = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} p (\langle n | \hat{M}_{11}^k(t) | n \rangle - \langle n | \hat{M}_{22}^k(t) | n \rangle), \tag{99}$$

$$\langle \hat{p}(t)^2 \rangle = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} p^2 (\langle n | \hat{M}_{11}^k(t) | n \rangle - \langle n | \hat{M}_{22}^k(t) | n \rangle), \tag{100}$$

where $\langle n | \hat{M}_{11}^k(t) | n \rangle$ and $\langle n | \hat{M}_{22}^k(t) | n \rangle$ are given by (84) and (85), respectively.

In Figs. 3a–c we have plotted $\Delta p(t)$ for $\vec{q} \cdot \vec{g} = 0$, $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$ and $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$, respectively. In Fig. 3a the Rabi-like oscillations can be identified, but in Figs. 3b and c, when the influence of the gravitational field increases, the Rabi oscillations disappear and the atomic momentum diffusion becomes positive and the atom does not recoil, because before absorption of photon, the atom is deflected by the gravitational field. Moreover, the atom can experience larger light-induced forces during its interaction with the radiation field, when the gravitational field increases.

4.4 Photon Counting Statistics

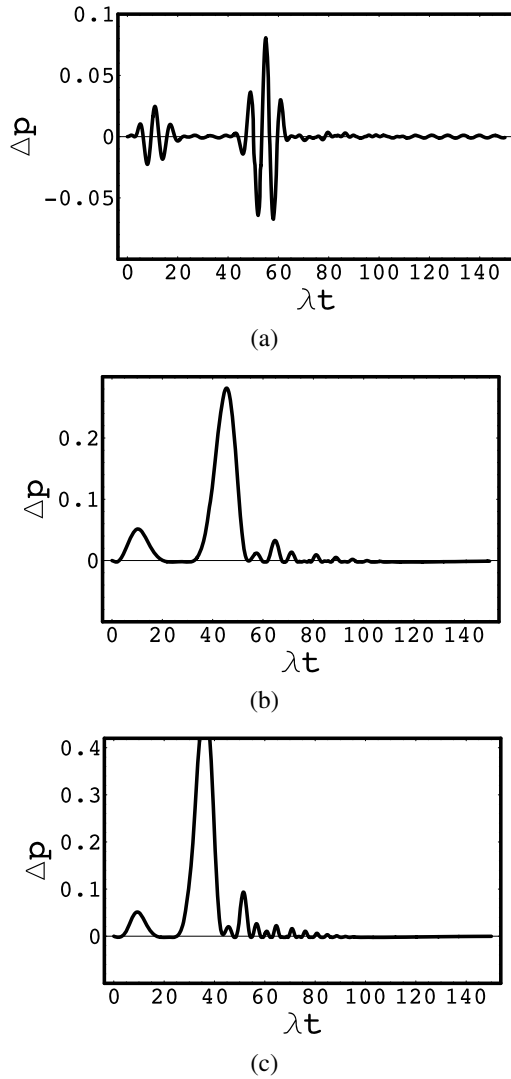
One of the most remarkable nonclassical effects is the sub-Poissonian photon statistics of the radiation field. To determine the influence of gravity on such effect in the presence of phase damping we consider the Mandel Q parameter defined by [52, 53]

$$Q(t) = \frac{(\langle n(t)^2 \rangle - \langle n(t) \rangle^2)}{\langle n(t) \rangle} - 1. \tag{101}$$

For $-1 \leq Q < 0$ ($Q > 0$), the statistics is sub-Poissonian (super-Poissonian); $Q = 0$ stands for Poissonian statistics. We have

$$Q(t) = \left(\left\{ \left[\sum_{n=0}^{\infty} n^2 P(n, t) \right] - \left[\sum_{n=0}^{\infty} n P(n, t) \right]^2 \right\} \left[\sum_{n=0}^{\infty} n P(n, t) \right]^{-1} \right) - 1, \tag{102}$$

Fig. 3 Time evolution of the atomic momentum diffusion versus the scaled time λt with the same corresponding data used in Fig. 1: **a** For $\vec{q} \cdot \vec{g} = 0$; **b** For $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$; **c** For $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$



where the probability of finding n photons in the radiation field is found to be

$$P(n, t) = \langle n | \hat{\rho}_{\text{field}}(t) | n \rangle = \langle n | \text{Tr}_{\text{atom}} \hat{\rho}_s(t) | n \rangle, \tag{103}$$

and by using (75) we obtain

$$P(n, t) = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n | \hat{M}_{11}^k(t) | n \rangle + \langle n | \hat{M}_{22}^k(t) | n \rangle). \tag{104}$$

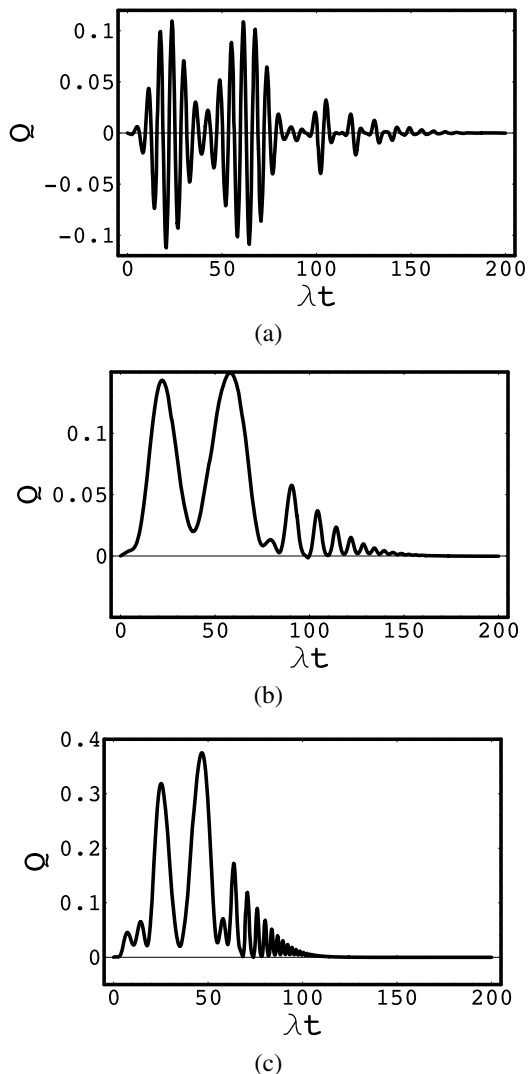
Therefore, by using (102) and (104) we obtain

$$Q(t) = \left(\left[\sum_{n=0}^{\infty} n^2 \left(\int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n | \hat{M}_{11}^k(t) | n \rangle + \langle n | \hat{M}_{22}^k(t) | n \rangle) \right) \right] \right)$$

$$\begin{aligned}
 & - \left[\sum_{n=0}^{\infty} n \left(\int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n | \hat{M}_{11}^k(t) | n \rangle + \langle n | \hat{M}_{22}^k(t) | n \rangle) \right) \right]^2 \Big\} \\
 & \times \left[\sum_{n=0}^{\infty} n \left(\int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n | \hat{M}_{11}^k(t) | n \rangle + \langle n | \hat{M}_{22}^k(t) | n \rangle) \right) \right]^{-1} - 1. \quad (105)
 \end{aligned}$$

The numerical results for three values of the parameter $\vec{q} \cdot \vec{g}$ are shown in Fig. 4. As it is seen, in the presence of phase damping the cavity-field exhibits alternately sub-Poissonian and super-Poissonian statistics when the influence of the gravitational field is negligible. With increasing $\vec{q} \cdot \vec{g}$ the sub-Poissonian characteristic is suppressed and the cavity-field exhibits super-Poissonian statistics. After some time, the Mandel parameter Q is stabilized

Fig. 4 Time evolution of the Mandel parameter $Q(t)$ versus the scaled time λt with the same corresponding data used in Fig. 1: **a** For $\vec{q} \cdot \vec{g} = 0$; **b** For $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$; **c** For $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$



at an asymptotic zero value; the larger the parameter \vec{q}, \vec{g} is more rapidly $Q(t)$ reaches the asymptotic value zero.

4.5 Quadrature Squeezing of the Cavity-Field

Squeezed states in quantum optics are characterized by reduced quantum fluctuations in one quadrature of the field at the expense of the increased fluctuations in the other quadrature. To analyze the squeezing properties of the phase damped radiation field in the presence of gravity, we introduce two slowly varying quadrature operators

$$\hat{X}_1(t) = \frac{1}{2}(\hat{a} \exp(i\omega t) + \hat{a}^\dagger \exp(-i\omega t)), \tag{106}$$

and

$$\hat{X}_2(t) = \frac{1}{2i}(\hat{a} \exp(i\omega t) - \hat{a}^\dagger \exp(-i\omega t)). \tag{107}$$

Squeezing is said to exist whenever $\langle(\Delta \hat{X}_i)^2\rangle \equiv \langle \hat{X}_i^2 \rangle - \langle \hat{X}_i \rangle^2 < \frac{1}{4}$, ($i = 1, 2$). In order to characterize the quadrature squeezing in the model under consideration we calculate the squeezing parameter S_i ($i = 1$ or 2) defined by

$$S_i(t) = 4\langle(\Delta \hat{X}_i(t))^2\rangle - 1, \tag{108}$$

where $-1 \leq S_i < 0$ for squeezing. In terms of the photon annihilation and creation operators, \hat{a} and \hat{a}^\dagger we have

$$S_1(t) = (\langle \hat{a}^2(t) \rangle - \langle \hat{a}(t) \rangle^2) \exp(2i\omega t) + (\langle \hat{a}^{\dagger 2}(t) \rangle - \langle \hat{a}^\dagger(t) \rangle^2) \exp(-2i\omega t) + 2(\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle - \langle \hat{a}^\dagger(t) \rangle \langle \hat{a}(t) \rangle), \tag{109}$$

and

$$S_2(t) = -(\langle \hat{a}^2(t) \rangle - \langle \hat{a}(t) \rangle^2) \exp(2i\omega t) - (\langle \hat{a}^{\dagger 2}(t) \rangle - \langle \hat{a}^\dagger(t) \rangle^2) \exp(-2i\omega t) + 2(\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle - \langle \hat{a}^\dagger(t) \rangle \langle \hat{a}(t) \rangle), \tag{110}$$

where $\langle \hat{a}(t) \rangle$, $\langle \hat{a}^\dagger(t) \rangle$ and $\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle$ are given by

$$\langle \hat{a}(t) \rangle = \text{Tr}_{\text{field}}(\hat{\rho}_{\text{field}}\hat{a}(t)), \quad \langle \hat{a}^\dagger(t) \rangle = \text{Tr}_{\text{field}}(\hat{\rho}_{\text{field}}\hat{a}^\dagger(t)), \tag{111}$$

$$\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle = \text{Tr}_{\text{field}}(\hat{\rho}_{\text{field}}\hat{a}^\dagger(t)\hat{a}(t)). \tag{112}$$

Therefore by using (75) we have

$$\langle \hat{a}(t) \rangle = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n | \hat{M}_{11}^k(t) \hat{a} | n \rangle + \langle n | \hat{M}_{22}^k(t) \hat{a} | n \rangle) = \langle \hat{a}^\dagger(t) \rangle^*, \tag{113}$$

$$\langle \hat{a}(t)^2 \rangle = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} (\langle n | \hat{M}_{11}^k(t) \hat{a}^2 | n \rangle + \langle n | \hat{M}_{22}^k(t) \hat{a}^2 | n \rangle) = \langle \hat{a}^{\dagger 2}(t) \rangle^*, \tag{114}$$

$$\langle \hat{a}^\dagger(t)\hat{a}(t) \rangle = \int d^3 p \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} n (\langle n | \hat{M}_{11}^k(t) | n \rangle + \langle n | \hat{M}_{22}^k(t) | n \rangle) \tag{115}$$

where

$$\begin{aligned}
 \langle n | \hat{M}_{11}^k(t) \hat{a} | n \rangle &= \sqrt{n} g_+^k(n, t) g_+^k(n-1, t) \psi_1(n, t) \psi_1^*(n-1, t) \\
 &\quad + \sqrt{n(n+1)} v_+^k(n+1, t) g_+^k(n-1, t) \psi_2(n+1, t) \\
 &\quad \times \psi_1^*(n-1, t) + n v_+^k(n, t) g_+^k(n, t) \psi_1(n, t) \psi_2^*(n, t) \\
 &\quad + n \sqrt{n+1} v_-^k(n+1, t) v_-^k(n, t) \psi_2(n+1, t) \psi_2^*(n, t), \tag{116}
 \end{aligned}$$

$$\begin{aligned}
 \langle n | \hat{M}_{11}^k(t) \hat{a}^2 | n \rangle &= \sqrt{n(n-1)} g_+^k(n, t) g_+^k(n-2, t) \psi_1(n, t) \\
 &\quad \times \psi_1^*(n-2, t) + \sqrt{n(n-1)(n+1)} v_+^k(n+1, t) g_+^k(n-2, t) \\
 &\quad \times \psi_2(n+1, t) \psi_1^*(n-2, t) + (n-1) \sqrt{n} v_-^k(n-1, t) g_+^k(n, t) \\
 &\quad \times \psi_1(n, t) \psi_2^*(n-1, t) + (n-1) \sqrt{n(n+1)} v_-^k(n+1, t) \\
 &\quad \times v_-^k(n-1, t) \psi_2(n+1, t) \psi_2^*(n-1, t), \tag{117}
 \end{aligned}$$

$$\begin{aligned}
 \langle n | \hat{M}_{22}^k(t) \hat{a} | n \rangle &= \sqrt{n} g_-^k(n, t) g_-^k(n-1, t) \psi_2(n, t) \psi_2^*(n-1, t) \\
 &\quad + \sqrt{n(n-1)} v_+^k(n-1, t) g_+^k(n, t) \psi_2(n, t) \\
 &\quad \times \psi_1^*(n-2, t) + n v_-^k(n, t) g_-^k(n-1, t) \psi_1(n-1, t) \psi_2^*(n-1, t) \\
 &\quad + n \sqrt{n-1} v_-^k(n-1, t) v_-^k(n, t) \psi_1(n-1, t) \psi_2^*(n-2, t), \tag{118}
 \end{aligned}$$

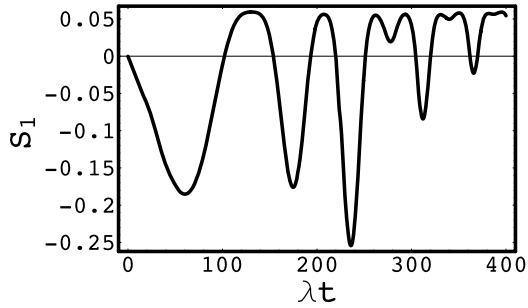
$$\begin{aligned}
 \langle n | \hat{M}_{22}^k(t) \hat{a}^2 | n \rangle &= \sqrt{n(n-1)} g_-^k(n, t) g_-^k(n-2, t) \psi_2(n, t) \psi_2^*(n-2, t) \\
 &\quad + \sqrt{n(n-1)(n-2)} v_+^k(n-2, t) g_-^k(n, t) \psi_2(n, t) \\
 &\quad \times \psi_1^*(n-3, t) + n \sqrt{n-1} v_-^k(n, t) g_+^k(n-2, t) \\
 &\quad \times \psi_1(n-1, t) \psi_2^*(n-2, t) + (n-1) \sqrt{(n-1)(n-2)} v_-^k(n, t) \\
 &\quad \times v_-^k(n-2, t) \psi_1(n-1, t) \psi_1^*(n-3, t). \tag{119}
 \end{aligned}$$

In Fig. 5 we have plotted the squeezing parameter $S_1(t)$ versus the scaled time λt for three values of the parameter $\vec{q} \cdot \vec{g}$ and a fixed value of the parameter γ . As it is seen, in the presence of phase damping the quadrature component \hat{X}_1 exhibits squeezing in the course of time evolution when the influence of the gravitational field is negligible. With increase of the parameter $\vec{q} \cdot \vec{g}$, the parameter S_1 shows damped oscillatory behavior and there is no quadrature squeezing.

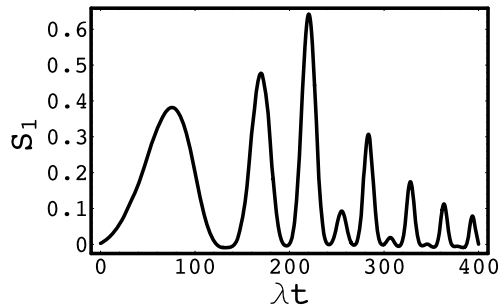
5 Summary and Conclusions

In this paper we have studied the dissipative dynamics of the phase damped JCM under the Markovian approximation in the presence of a classical homogeneous gravity field. The model consists of a moving two-level atom simultaneously exposed to the gravitational field

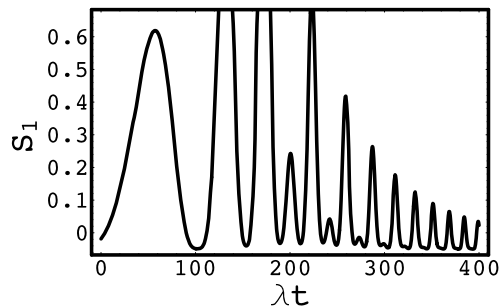
Fig. 5 Time evolution of the squeezing parameter $S_1(t)$ versus the scaled time λt with the same corresponding data used in Fig. 1: **a** For $\vec{q} \cdot \vec{g} = 0$; **b** For $\vec{q} \cdot \vec{g} = 0.5 \times 10^7$; **c** For $\vec{q} \cdot \vec{g} = 1.5 \times 10^7$



(a)



(b)



(c)

and a single-mode traveling radiation field in the presence of the phase damping. By making use of the super-operator technique, we have obtained an exact solution of the master equation for the density operator of the quantized atom-radiation system, under the Markovian approximation. Assuming that initially the radiation field is prepared in a coherent state and the two-level atom is in the excited state, we have investigated the influence of gravity on the temporal evolution of collapses and revivals of the atomic population inversion, atomic dipole squeezing, atomic momentum diffusion, photon counting statistics and quadrature squeezing of the radiation field in the presence of the phase damping. In particular, we have shown that the gravitational field suppresses nonclassical effects in the phase damped JCM. Indeed, in the model under consideration, both the damping parameter γ , which depends on the environment temperature and the gravity-dependent parameter $\vec{q} \cdot \vec{g}$ seriously reduce the quantum coherence. The approach adopted here may be applied to study various dissipa-

tive systems of two-level or three-level atoms interacting with quantized radiation field in a gravitational field. Finally, we want to point out that the method developed in this paper can be extended beyond the Markovian approximation. Work in this direction is in progress.

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